How General is Nilpotency?

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Abstract. Evidence is presented for the generality, criticality and importance of nilpotence and the associated criteria of Pauli exclusion, quantum phase factor and quantum holographic signal processing in relation to calculation, problem solving and optimum control of Heisenberg uncertainty in Quantum Interaction.

Nilpotent quantum mechanics

The nilpotent formulation of relativistic quantum mechanics / quantum field theory (Rowlands, 2007, 2008) is exceptionally compact and minimalistic, and for that reason particularly powerful. It is also unique in generating significant physical interpretations, not available to more restricted formalisms. In the nilpotent formalism, the only requirement for defining the entire quantum mechanical apparatus relating to a fermion and its interactions is to specify its creation operator in the form ($i\boldsymbol{k}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m$), where $\boldsymbol{k}$, $\boldsymbol{i}$, $\boldsymbol{j}$ are quaternion units and $\mathbf{p}$ takes the form of a multivariate or quaternion-like vector. Wavefunction, phase factor, amplitude, spinor structure, vacuum, and quantum mechanical equation are then automatic consequences of the initial definition and do not represent independent information.

While the operators E and $\mathbf{p}$ typically represent the derivatives $i\partial / \partial t$ and $-i\nabla$, they can also be supposed to incorporate field terms or be covariant derivatives, so that E could be, say, $i\partial / \partial t + e\phi + \ldots$, and $\mathbf{p}$ could be, say, $-i\nabla + e\mathbf{A} + \ldots$, with any number of field terms (whether electric, strong, weak or gravitational) added. The spinor structure is automatically provided by the four possible sign variations of E and $\mathbf{p}$, so that the full specification of the operator becomes, in abbreviated form, ($\pm i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m$), with the sign variations representing the terms in a 4-component row vector. Once specified, this operator must determine a *unique* phase factor on which it acts to produce an amplitude or wavefunction which is nilpotent or squares to zero, and the amplitude must be a 4-component column vector, again of the form ($\pm i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m$), but this time multiplied by the phase factor, with the E and $\mathbf{p}$ representing eigenvalues (though not necessarily constant ones) rather than derivatives.

The zeroing of this object by squaring then becomes the only quantum mechanical equation needed.

Physically, the nilpotency or squaring to zero may be interpreted as Pauli exclusion, as the combination state of two identical fermions cannot exist, but the operation also indicates the reason for Pauli exclusion, as being the need to maintain a zero totality universe. In effect, in creating a fermion from absolutely nothing in the form $(\pm i\mathbf{k}E \pm \mathbf{i}\mathbf{p} + \mathbf{j}m)$, we also create the *vacuum* or rest of the universe to which this fermion relates in the form $-(\pm i\mathbf{k}E \pm \mathbf{i}\mathbf{p} + \mathbf{j}m)$, and the combination state of fermion and vacuum or rest of the universe must be precisely $-(\pm i\mathbf{k}E \pm \mathbf{i}\mathbf{p} + \mathbf{j}m)(\pm i\mathbf{k}E \pm \mathbf{i}\mathbf{p} + \mathbf{j}m)$, or exactly zero, in the same way as the superposition. This is true whether the fermion is free or subject to any number of interaction potentials; in the latter case, vacuum or the 'rest of the universe' will be 'constructed' in such a way that the existence of a fermion in that state becomes possible. In this sense, every fermion contributes to the rest of the universe seen by every other fermion, and the 'universe' is defined by the state which makes all the nilpotent conditions possible simultaneously.

A nilpotent formulation of the fermion state means that the fermion can only be defined with respect to the entire universe, or, in another way of describing it, with respect to the entire quantum field. When we define energy conservation by writing $(\pm i\mathbf{k}E \pm \mathbf{i}\mathbf{p} + \mathbf{j}m)(\pm i\mathbf{k}E \pm \mathbf{i}\mathbf{p} + \mathbf{j}m) = 0$, we are saying that a closed system involving a fermion is impossible, that the conservation principle applies only over the whole universe, though, in being intrinsically relativistic, it is also local. Thermodynamics, as constructed in the first two laws, becomes a necessary consequence; the fermion is a fundamentally dissipative system, and this is required by its nilpotent structure.

Local conservation over the whole universe, which is effectively defined for each fermion from within the bracket $(\pm i\mathbf{k}E \pm \mathbf{i}\mathbf{p} + \mathbf{j}m)$, is an important principle. However, the Pauli exclusion aspect of nilpotency, which is defined from *outside* the bracket, is equally significant, and is nonlocal. Here, every fermion must be constructed in such a way that its bracketed $(\pm i\mathbf{k}E \pm \mathbf{i}\mathbf{p} + \mathbf{j}m)$ is different from any other, so that their product state is nonzero, and this must happen instantaneously, as there is no localization condition, as there is inside the bracket. So the state of every fermion in the universe must be subject to two universal conditions, one local and one nonlocal.

Conventionally, of course, Pauli exclusion between ψ_1 and ψ_2 is defined in terms of the antisymmetric wavefunction:

$$\psi_1\psi_2 - \psi_2\psi_1 = -(\psi_2\psi_1 - \psi_1\psi_2).$$

Applying the nilpotent condition to this leads to a truly remarkable result:

$$
\begin{aligned}
&(\pm i\boldsymbol{k}E_1 \pm \boldsymbol{i}\mathbf{p}_1 + \boldsymbol{j}m_1)\,(\pm i\boldsymbol{k}E_2 \pm \boldsymbol{i}\mathbf{p}_2 + \boldsymbol{j}m_2)\\
&-(\pm i\boldsymbol{k}E_2 \pm \boldsymbol{i}\mathbf{p}_2 + \boldsymbol{j}m_2)\,(\pm i\boldsymbol{k}E_1 \pm \boldsymbol{i}\mathbf{p}_1 + \boldsymbol{j}m_1)\\
&= 4\mathbf{p}_1\mathbf{p}_2 - 4\mathbf{p}_2\mathbf{p}_1 = 8\,i\,\mathbf{p}_1 \times \mathbf{p}_2.
\end{aligned}
$$

This result immediately implies that only the instantaneous direction of **p** ensures that the two nilpotent wavefunctions remain distinct, and that all the significant information about the state is contained in this term. Significantly, decoherence in combinations of more than one state is ultimately due to the vector nature of the **p** term.

Ultimately, the key significance of the nilpotent fermion state ($\pm i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m$) is that it necessarily constructs its vacuum, environment, or 'rest of the universe, as its own reversed image, $-(\pm i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$. The quantum mechanics, and the logic to which it relates, is fundamentally holistic. We have a simultaneous description, at all times, of the system and its environment. In addition, because we have derived it from a more fundamental information processing system, privileging zero totality, it is probable that that information processing system has applications beyond the narrow confines of quantum mechanics.

The nilpotent supersymmetry

Nilpotency is a particularly powerful idea because it introduces a natural holism into the definition of any conservative system. Clearly, some kind of holistic principle is necessary to make sense of the way that nature behaves, in structuring its operations as though all events had a universal and nonlocally determined unique birthordering. Richard Feynman once wrote: 'It always bothers me that, according to the laws as we understand them today, it takes a computing machine an infinite number of logical operations to figure out what goes on in no matter how tiny a region of space, and no matter how tiny a region of time. How can all that be going on in that tiny space? Why should it take an infinite amount of logic to figure out what one tiny piece of spacetime is going to do?' (Feynman, 1965) In fact, neither the brain nor any other natural system works like this. There is always a holistic process.

Nilpotent logic rather than digital logic reflects this by making the universal automatically the mirror image of the particular because the universe is constrained to have zero totality. This clearly operates in the case of quantum mechanics. The question that then emerges is how much can any system (e.g. life, consciousness, galactic formation, chemistry), which has a strong degree of self-organization manage to achieve this by being modeled on a nilpotent structure. The work of Hill and Rowlands (2007), and Marcer and Rowlands (2007), suggests that this is possible in a wide variety of contexts. The reason is that the nilpotency does not stem

from quantum mechanics initially, but from fundamental conditions of optimal information processing which are prior to physics, chemistry and biology, and even to mathematics.

The work of Vitiello (2008) suggests a striking confirmation outside the authors' own work (Marcer and Rowlands, 2007). Vitiello has developed a quantum field model of the brain in which dissipation doubles the degrees of freedom of the system because the environment acts like the system's time-reversed mirror image. Essentially energy $-E$ dissipated by the system is balanced by energy E absorbed by the environment, and the two are structurally identical, though opposite in sign. This is an almost exact description of what happens in a nilpotent system. He also says that: 'Consciousness appears to be rooted in the permanent dialog of the subject with its Double.' Nilpotency provides a mechanism by which this can be achieved.

Nilpotence is the unique criterion of the universal computational rewrite system (NUCRS) set out in *Zero to Infinity* (Rowlands, 2007). It specifies a novel evolutionary worldview in terms of a staircase of matter of increasing complexity. This nilpotent staircase, obtained from the spontaneous symmetry breaking of the empty state taken as the worldview's initial nilpotent boundary condition (essential to the proper solution of any physical problem), begins by predicting the quantizations of Standard Model elementary particle matter as experimentally validated. It describes these (particles) as the sources and sinks of the 3 + 1 relativistic space time field, in such a way that the field and its sources and sinks operationally constitute the quantum computational 'machine order code' for all further (universal rewrite) computation corresponding to the NUCRS infinite universal alphabet and grammar. A field, Rowlands (2008, 2009) shows, concerns relativistic quantum mechanics derived from a single operator.

In principle, therefore, this nilpotent computational worldview is able to rewrite physical law at each level or stair in terms of the laws at the previous levels beginning from that of the initial 'machine order code' law. For example. it has already been shown (Hill and Rowlands, 2008) that the DNA / RNA genetic code is almost certainly a complete nilpotent rewrite of the NUCRS of the Standard Model of the elementary particles, rewritten at a molecular level, where the nilpotent worldview predicts the molecular biological structures and their molecular biology as they are actually known. And it generalizes molecular biology because the genetic code is now not only a unique, universal computational code but also a semantic one, as the NUCRS shows, of in principle the relativistic 3D geometry of its organisms in quantum holographic encoded form.

The symmetries of 3D space as a central feature of the NUCRS nilpotent QM machine order code and grammar

Consideration of the set of radial vectors to the surface of a unit sphere centre P, as defining each neighbourhood in 3D space, shows that any such vector through the centre *P* of the sphere and the plane perpendicular to it, are unique and correspond to the symmetries, *U*(1), *SU*(2), in relation to the symmetry of the sphere *SU*(3). Each *P* thus defines a ray space (Fig. 1), where the fundamental quantum mechanical spectral theorem of Hilbert and Von Neumann applies, such that *U*(1), *SU*(2) and *SU*(3) are respectively the symmetry groups, now known to determine the quantizations of the elementary particles of the electromagnetic, weak and strong forces, and $U(1) \times SU(2)$ is the symmetry of the electroweak force (the photon with its vector bosons) combined in just the right way. Thus (subject to the proviso that every *P* is indeed a Lie topological neighbourhood, where the symmetries are those of Lie groups together with their Lie algebras, as defined by their smooth tangent spaces of lines and planes) the NUCRS grammatical sub-alphabet of *P*, which must differentiate lexicographically between the three axes of space in order to describe three distinct orthogonal spatial axes, may be labelled ‘electron’, ‘muon’ and ‘tau’, in good accord with the Pauli exclusion principle for spin ½ (fermion) anti-commutative states, which applies to each and every unique unit vector and its two perpendicular plane axes at *P*.

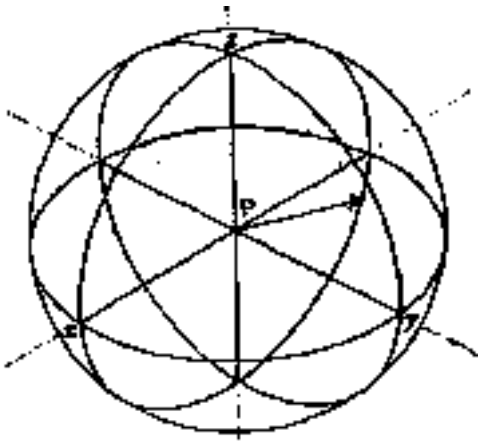

Fig. 1. 3D ray space at point *P*

In other words, in the NUCRS, where the elementary particles predicted are exclusively those of the Standard Model, and 3D space corresponds, as described above, to a quantum field, the particles with spin ½, known as the neutrinos take on, in a new testable hypothesis of their behaviour, an unexplored role in respect to the axes of 3D space. And in respect of this ‘3D spatial quantum field’, there exists a Lie symmetry, that

of the 3D Heisenberg Lie Group G, which in the form of its nilpotent Lie algebra g, as was known to Weyl in 1928, defines the Heisenberg uncertainty, as a Robertson relation,

$$\Delta U_\nu(P).\Delta U_\nu(Q) \geq ½ |U_\nu(Z)|$$

expressed in terms of standard root mean square deviations of the operators $\Delta U_\nu(P)$ and $\Delta U_\nu(Q)$, where $\{P, Q, Z\}$, defined below, are the canonical basis of g, and U_ν frequency ν, are, up to a unitary isomorphism, unique infinite dimensional irreducible unitary linear representations of G of the Schrodinger type in the standard Hilbert space $L^2(R)$ (Schempp, 1992). However as both g and G have Lie duals, there exist corresponding (Lie) exponential differentiable mappings with differentiable inverses, so that 'Heisenberg uncertainty' can, in this case, be used as the actual means to compute geometrically. As, for example, the description of $U(1, C)$ signal processing in the form of quantum holographic encoding / decoding in Magnetic Resonance Imaging (MRI) machines proves (Schempp, 1998).

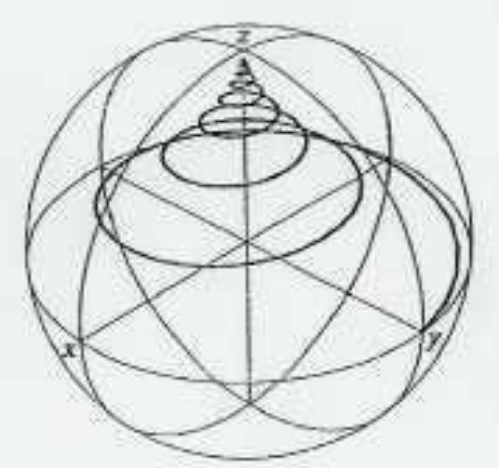

Fig. 2. Quantum wave collapse and re-expansion

Fig. 2 illustrates how in actual 3D space, the encoding / decoding Fourier transform action (in accord with the Heisenberg uncertainty principle defined by g the Lie algebra of G) actual happens in MRI. It shows the 'frequency induced signal' $U(1,C)$ described by the Heisenberg helix of G off resonance losing amplitude (z axis), i.e. thermodynamically decaying due to a transverse relaxation effect, but, remarkably, simultaneously regaining energy due to longitudinal relaxation, so as to embed the $U(1)$ signal in the complex plane $C = (x + iy)$, as a phase difference. For, with respect to G and g, G represented as the matrix

$$\begin{pmatrix} 1 & x & z \\ 0 & 1 & y \\ 0 & 0 & 1 \end{pmatrix}$$

is such that (x – path difference, y – phase difference) form a Fourier duality encoding pair $(x, y) = (x + iy)$ so embedding the complex plane C in G, and the infinitesimal Lie generators $\{P, Q, Z\}$ of g, where $(P, Q) = Z$; $(P, Z) = 0$; $(Q, Z) = 0$, are represented by the (3 line) matrices

$$P=\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; Q=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}; Z=\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

having Lie exponential diffeomorphisms

$$\exp P=\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}; \exp Q=\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}; \exp Z=\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

are the components of the automorphism of G, able to describe the inverse Fourier transform action of decoding, i.e. they give rise to the identity $\exp(xP + yQ + (z - ½xy)Z) = G$, where the conformal complex mode coordinates $T = ½(P + iQ)$ and $T^* = ½(P – iQ)$ permit through the linear Schrodinger representation U of G, a quantum mechanical description at the photon level in terms of the creation / annihilation operators $a = U(T)$ and $a^* = U(T^*)$ of an emitter/absorber model. Thus, for example, optimal control of quantum interactions in real time becomes possible as already experimental demonstrated in chemical reactions, equivalent to the solution of the Schrodinger equation for the chemistry (Dahleh et al, 1990, Judson and Rablitz, 1992, Rice, 1992).

Support for nilpotence from the Standard Model

The evidence from MRI presented in diagram B as specified above therefore explains the dichotomy between the facts of Heisenberg uncertainty and that quantum mechanical theory allows no loss of quantum mechanical information as is in principle the case in the quantum holographic process. at the nilpotent point where the collapse of the wave function, its annihilation turns into a re-expansion, a creation. However Diaz and Rowlands (2006), have shown that the NUCRS infinite alphabet

determined by the nilpotents $X_n^2 = 0$, $X_n \neq 0$ corresponds to the infinite square roots of –1, $n = 1, 2, \ldots$. This is in line with Berry's hypothesis (1986), that the imaginary parts of the non trivial zeros of the Riemann zeta function are the eigenvalues of some still unknown self adjoint Hamiltonian operator with time reversal asymmetry (a property of the NUCRS) of which phase space trajectories are chaotic.

It is in agreement with the fact the each letter of the NUCRS infinite alphabet has the same 'nilpotent' formulation $X_n^2 = 0$ at all levels of its rewrite structure and so is self similar of fractal dimension 2 and therefore embeddable in the complex plane C; i.e. can be mapped in a conformal fashion onto the open unit disc (which is the geometry of $i = \sqrt{-1}$). It thus corresponds to the universal fractal attractor of the Golden number (Fig. 3) and relates to the wave behaviour seen at the boundary of the Mandelbrot set.

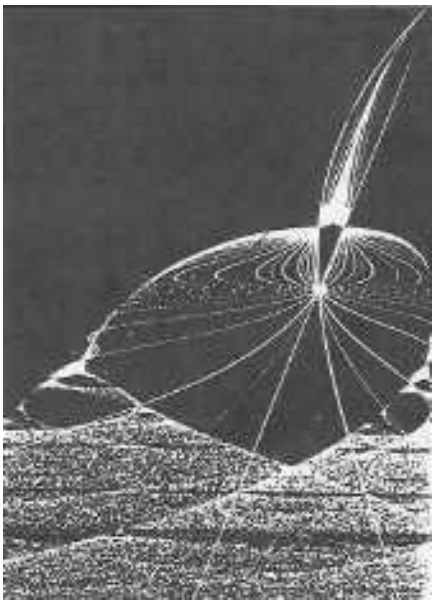

Fig. 3. The universal Golden attractor

The harmonic analysis on the 3D Heisenberg Lie group G, discovered and applied by Schempp (1986) to the description and control of MRI and Synthetic Aperture Radars, proves is indeed be the case. So also do the field symmetry $U(1, C)$ in terms of the polarization wave property of electromagnetic signals in relation, for example, to the quantum mechanical Aharonov Bohm effect; the helices of RNA / DNA (Hill and Rowlands, 2008) and the representation the Riemann sphere of the complex plane by stereographic projection, where the pole is the point at infinity called in perspective the 'vanishing point', which in art is a very late discovery of the nature of human perception (Fig. 4).

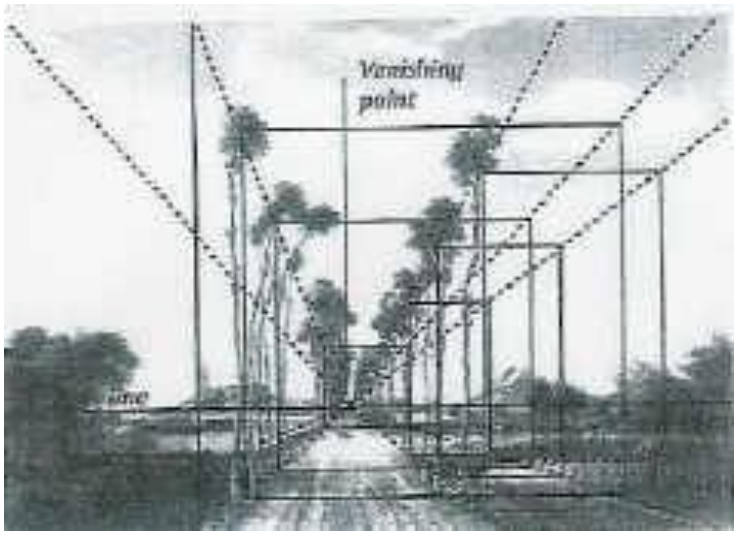

Fig. 4. Perspective with vanishing point / pole

In further support of this NUCRS hypothesis, Trell (2008) uses the 'cubit' symmetry structure of the eight cubes about the point P of the neighbourhood with its eightfold way cubit metric scaling of signature +++, –++, +–+, ++–, +—, –+–, —+, —— to calculate the relative masses of all the families of the elementary particles, in good accord with their experimentally determined values, and their quantizations as previously calculated by Rowlands. Mathematically these properties are, of course, those to be expected from Lie transformational theory, where bi-linearisation takes place via the calculation of invariants. A fact that would be explained if these invariants are all properties of 3D universal standing wave or a soliton.

The very highly detailed paper by Hill and Rowlands (2008) in relation to DNA / RNA biological systems and their geometric structures, Nature's Code fully underpins and extends that from the earlier complementary perspective of the nilpotent Heisenberg Lie group *G* (Marcer and Schempp, 1986, Gariaev et al, 2001, 2002), which predict, for example, the structure of DNA as a semantic wave bio-computer where the two Heisenberg helices of *G* and its dual *G*' and their base-pairing SU(2) hologram planes are the basis for the U(1, C) holographic helical signal processing taking place and the semiotics of the genetic code. This very comprehensive treatment fully substantiates the NUCRS hypothesis that the DNA / RNA genetic code is indeed a rewrite at a higher level of molecular complexity further extending the NUCRS universal alphabet. And where the DNA is known to include the encoding of the human brain at a yet higher level (Marcer and Rowlands, 2007). To summarize therefore. 3D space should no longer be considered as just a plenum within which matter moves, but as a quantum field such as Einstein

describes in general relativity, where, it has been said, 'matter bends space and space shapes matter'. An example would be changes in scaling resulting from nearby massive objects that can be expected from Trell's research to subject $U(1)$ signals to lensing effects. Indeed as shown in *Zero to Infinity* (Rowlands, 2007), this is, in fact, the actual means by which the dichotomy between nilpotent quantum mechanics and general relativity is resolved thermodynamically.

Further support, at a higher level of complexity, comes from the three spin ½ particles, the electron, muon and tau, and the composite (nuclear) matter with spin – these matter symmetries would (in addition to the Trell metric signature of 'cubit' scaling) reflect those of $SU(3)$, which concern the quark patterns, where these are, rather than the doublets of $SU(2)$, expressed in the form of triplets conventionally labelled the red, yellow and blue colour forces interactions produced by gluons (colour bosons) between the quarks.

It is also clear that in relation to the 3D spatial field, that there must also exist $SU(2)$ processes able to change the properties of $SU(3)$ matter interior to any nuclear composite. These processes can thus be hypothesized to define, for example, the two very well known phenomena of nuclear fusion and nuclear fission (as the two NUCRS create, conserve productions at this level) where the interactions involve the neutrinos and are recognised to produce the composite nuclear matter/elements of the periodic table in the stars, which are seen here to be 'super' quantum mechanical ray spaces composed from neighbourhoods P.

The NUCRS / nilpotent quantum mechanics (NQM) scaling phenomena and relativity – special and general – are essentially one and the same and so relate to the fact that the electro-magnetic vector symmetry $U(1)$ and 'gravity' / scaling, can be expected in the first approximation to obey the same inverse square Coulomb law as is indeed in the case. For in the case of gravity masses replace charges, but due to their scalar rather than vector nature only attract one another.

Conclusion

Evidence from phenomena at various levels of complexity seems to suggest that a nilpotent structure providing an automatic determination of the environment simultaneously with the system is the most efficient and most ubiquitous information processing system in Nature, and acts, in effect, as Nature's machine order code.

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